

Technical report  
Sparse Principal Component Solver

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## 0.1 Sparse Principal Component Problem

A formulation of the Sparse Principal Component Problem is as follows

$$(\text{SPCA}_{\Sigma}^k) \begin{cases} \max & x^T \Sigma x \\ \text{s.t.} & x^T x = 1, \\ & \|x\|_0 \leq k, \\ & x \in \mathbb{R}^n, \end{cases} \quad (1)$$

where the integer parameter  $1 \leq k \leq n$  controls the sparsity of the solution.

## 0.2 Extracting several sparse components

A well know technique to extract more than one sparse component is by deflating the covariance matrix at each round. see Mackey et al.

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**Algorithm 1** Generalized Deflation Method for Sparse PCA

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- 1: Given: covariance matrix  $\Sigma_0$ , number of rounds  $r$ ,  $\{k_1, \dots, k_r\} \subset \{1, \dots, n\}$ .
  - 2:  $B_0 := I$ ,
  - 3: **for**  $t = 1, \dots, r$  **do**
  - 4:    $x_t := \underset{x^T B_{t-1} x = 1, \|x\|_0 \leq k}{\operatorname{argmax}} x^T \Sigma_{t-1} x$
  - 5:    $q_t := B_{t-1} x_t$
  - 6:    $\Sigma_t := (I - q_t^T q_t) \Sigma_{t-1} (I - q_t^T q_t)$
  - 7:    $B_t := B_{t-1} (I - q_t^T q_t)$
  - 8:    $x_t := x_t / \|x_t\|_2$
  - 9: **end for**
  - 10: **return**  $\{x_1, \dots, x_r\}$
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An approach for solving point 4: in Algorithm 1 is to first replace  $x^T B_{t-1} x = 1$  by  $x^T (B_{t-1} + \epsilon I) x = 1$  and use Woodbury matrix identity to transform it in the form of 1. To be detailed!

### 0.3 Implemented methods

- Dca method from Thiao et al.
- Gspca forward and backward methods from Moghaddam et al.
- We will implement all the sparse principal component methods and deflation techniques and incorporate them into the solver.

# Bibliography

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- [3] Moghaddam, B., Weiss, Y., & Avidan, S.: *Spectral bounds for sparse PCA: Exact and greedy algorithms*. Advances in Neural Information Processing Systems (2006), 18.